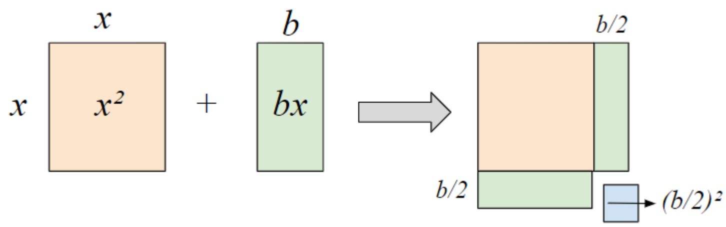
Math 1320: Quadratic Functions in Vertex Form

What is completing the square? Completing the square is a method for rewriting a quadratic function in vertex form:

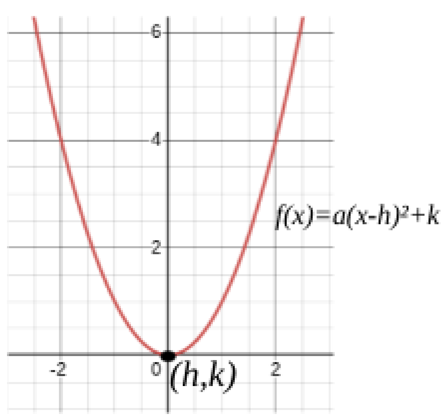
Quadratic Function	$\Rightarrow$	Vertex Form
$f(x) = ax^2 + bx + c$	Complete the Square	$f(x) = a(x-h)^2 + k$

Consider the expression $x^2 + bx$. The diagram below shows what's happening geometrically when we complete the square:



When we rearrange $x^2 + bx$, it almost creates a square. We are just missing a small square with the area of $(\frac{b}{2})^2$. Below is the process for completing the square algebraically:

Why is completing the square important? Completing the square will be used to rewrite quadratic functions in vertex form. Vertex form tells us how the parabola opens and the location of the vertex (h, k). The vertex is the minimum or maximum value of a quadratic function. These characteristics are helpful when graphing quadratic functions by hand, a skill we will use in this course.



What questions may I be asked that use completing the square? Many applications involve finding the maximum or minimum value of a quadratic function and where the value occurs. For this reason, we will need to use completing the square to put a given quadratic function in vertex form.

Example 1. Vertex Form
Rewrite the quadratic function in vertex form:

$$f(x) = -x^2 + 10x - 21$$

Example 2. Application
An athlete shoots a free throw in a basketball game. The athlete shoots the ball from a height of 6 feet. The height of the ball, $f(x)$, in feet, can be modeled by

$$f(x) = -0.1x^2 + 1.6x + 6$$

where x is the ball's horizontal distance, in feet, from where it was thrown.

1. What is the maximum height of the ball and how far from the free throw line does this occur?

Practice Problems

Practice finding the vertex of the following quadratic functions. State whether the vertex point is a maximum or a minimum.

1. $f(x) = x^2 + 2x - 3$ $\quad$ [$(-1, -4)$, Minimum]
2. $g(x) = 2x^2 - 8x + 6$ $\quad$ [$(2, -2)$, Minimum]
3. $h(x) = -x^2 - 2x + 6$ $\quad$ [$(-1, 7)$, Maximum]